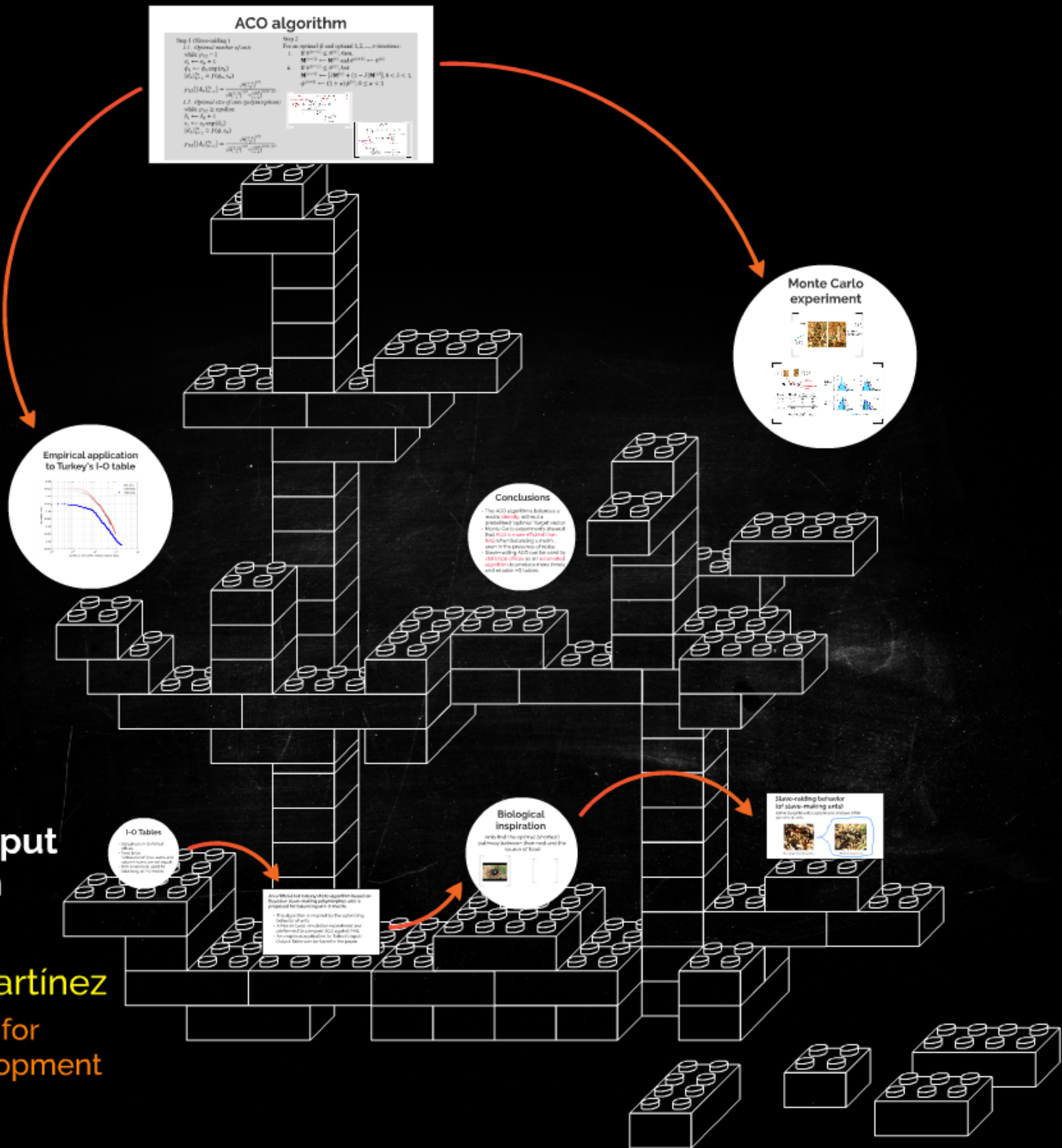

Balancing Input-Output tables with Bayesian Slave-raiding Ants

Rolando Gonzales Martínez

Bayesian Institute for Research & Development

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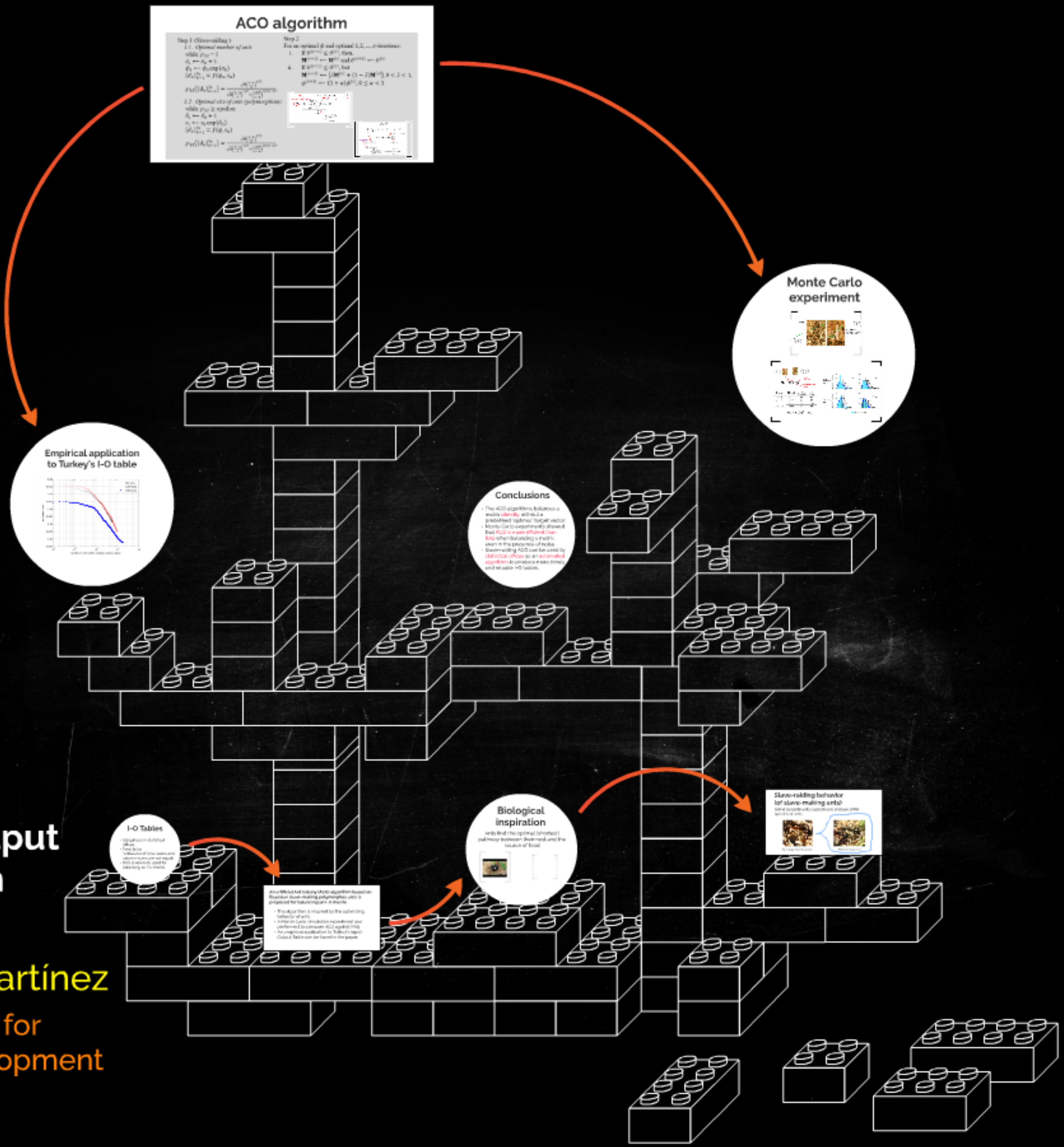


I-O Tables

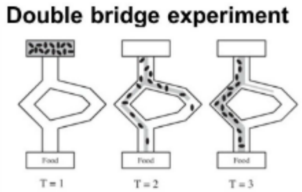
- Ubiquitous in statistical offices.
- Tend to be "unbalanced" (row-sums and column-sums are not equal).
- RAS is normally used for balancing an I-O matrix.

An artificial Ant Colony (ACO) algorithm based on Bayesian slave-making polymorphus ants is proposed for balancing an I-O matrix.

- The algorithm is inspired by the optimizing behavior of ants.
- A Monte Carlo simulation experiment was performed to compare ACO against RAS.
- An empirical application to Turkey's Input-Output Table can be found in the paper.

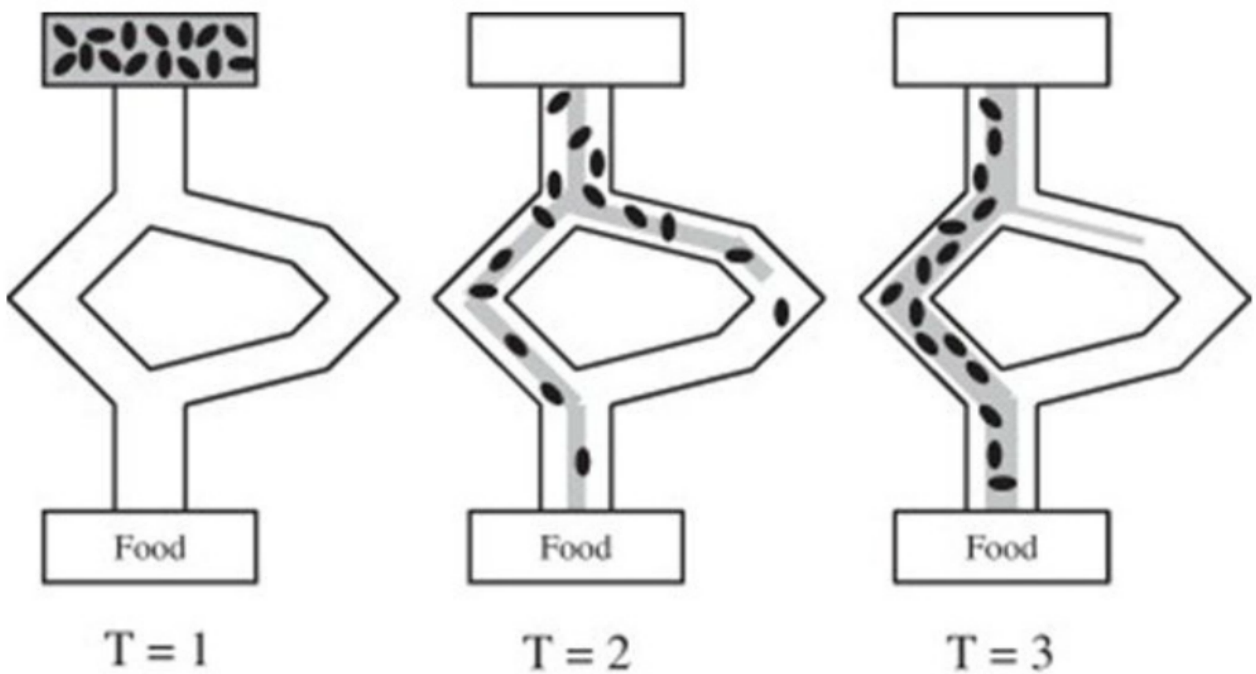
Biological inspiration

Ants find the optimal (shortest) pathway between their nest and the source of food




YouTube

Double bridge experiment

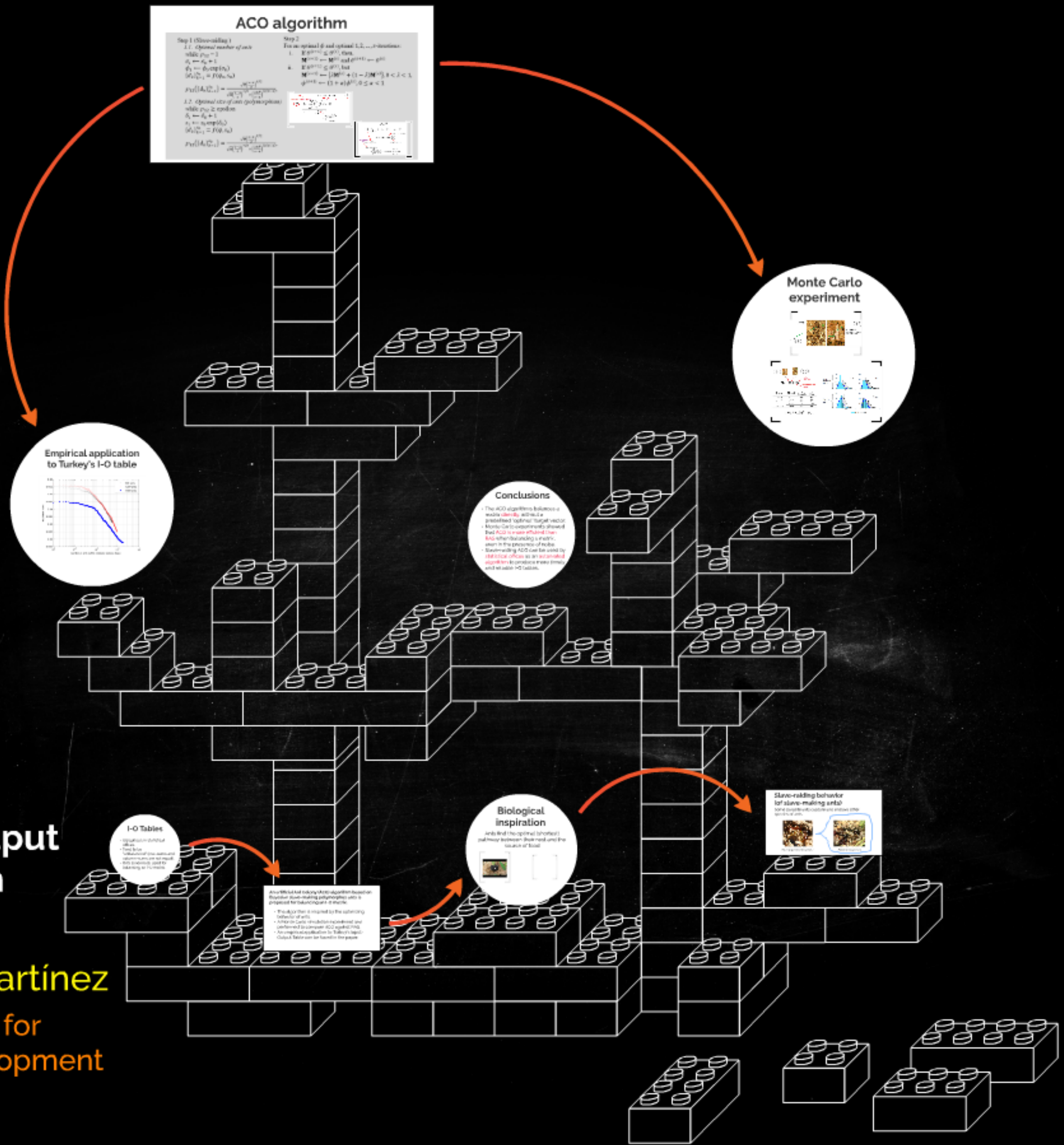


Slave-raiding behavior (of slave-making ants)

Some parasite ants capture and enslave other species of ants

Rossomyrmex minuchae

Proformica ferreri



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ACO algorithm

Step 1 (Slave-raiding)

1.1. Optimal number of ants

while $\mathcal{p}_{12} = 1$

$\sigma_1 \longleftarrow \sigma_0 + 1$

$\phi_1 \longleftarrow \phi_0 \exp(\sigma_0)$

$\{d_k\}_{k=1}^{s_0} = f(\phi_0, s_0)$

$$\mathcal{p}_{12}\left(\{d_k\}_{k=1}^{s_0}\right) = \frac{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)}},$$

1.2. Optimal size of ants (polymorphism)

while $\mathcal{p}_{12} \geq$ epsilon

$\delta_1 \longleftarrow \delta_0 + 1$

$s_1 \longleftarrow s_0 \exp(\delta_0)$

$\{d_k\}_{k=1}^{s_0} = f(\phi, s_0)$

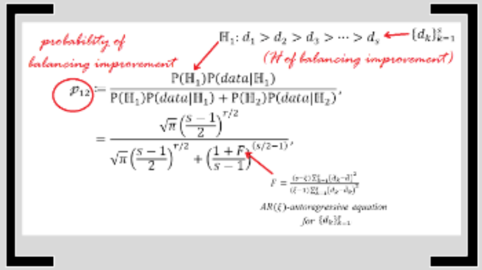
$$\mathcal{p}_{12}\left(\{d_k\}_{k=1}^{s_0}\right) = \frac{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)}},$$

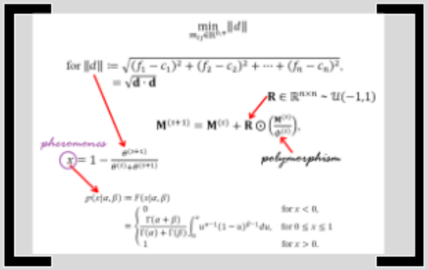
Step 2

For an optimal ϕ and optimal $1, 2, \ldots, s$-iterations:

i. If $\theta^{(s+1)} \leq \theta^{(s)}$, then,
$\mathbf{M}^{(s+1)} \longleftarrow \mathbf{M}^{(s)}$ and $\theta^{(s+1)} \longleftarrow \theta^{(s)}$

ii. If $\theta^{(s+1)} \leq \theta^{(s)}$, but
$\mathbf{M}^{(s+1)} \longleftarrow \left\{\lambda \mathbf{M}^{(s)} + (1-\lambda)\mathbf{M}^{(s)}\right\}, 0 < \lambda < 1,$
$\phi^{(s+1)} \longleftarrow (1+\alpha)\phi^{(s)}, 0 \leq \alpha < 1$





$$\{d_k\}_{k=1}^{s}$$

$$\mathbb{H}_1: d_1 > d_2 > d_3 > \cdots > d_s$$

(H of balancing improvement)

probability of balancing improvement

$$p_{12} := \frac{\mathrm{P}(\mathbb{H}_1)\mathrm{P}(data|\mathbb{H}_1)}{\mathrm{P}(\mathbb{H}_1)\mathrm{P}(data|\mathbb{H}_1) + \mathrm{P}(\mathbb{H}_2)\mathrm{P}(data|\mathbb{H}_2)},$$

$$= \frac{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi}\left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)}},$$

$$F = \frac{(s-\xi)\sum_{k=1}^{s}\left(\hat{d}_k - \bar{d}\right)^2}{(\xi-1)\sum_{k=1}^{s}\left(d_k - \hat{d}_k\right)^2}$$

$AR(\xi)$-autoregressive equation for $\{d_k\}_{k=1}^{s}$

$$\min_{m_{ij} \in \mathbb{R}^{0,+}} \|d\|$$

$$\text{for } \|d\| := \sqrt{(f_1 - c_1)^2 + (f_2 - c_2)^2 + \cdots + (f_n - c_n)^2}, \\ = \sqrt{\mathbf{d} \cdot \mathbf{d}}$$

$$\mathbf{R} \in \mathbb{R}^{n \times n} \sim \mathcal{U}(-1,1)$$

$$\mathbf{M}^{(s+1)} = \mathbf{M}^{(s)} + \mathbf{R} \odot \left(\frac{\mathbf{M}^{(s)}}{\phi^{(s)}}\right),$$

polymorphism

pheromones

$$x = 1 - \frac{\theta^{(s+1)}}{\theta^{(s)} + \theta^{(s+1)}}$$

$$p(x|\alpha,\beta) := F(x|\alpha,\beta) = \begin{cases} 0 & \text{for } x < 0, \\ \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)+\Gamma(\beta)} \displaystyle\int_0^x u^{\alpha-1}(1-u)^{\beta-1} du, & \text{for } 0 \le x \le 1 \\ 1 & \text{for } x > 0. \end{cases}$$

ACO algorithm

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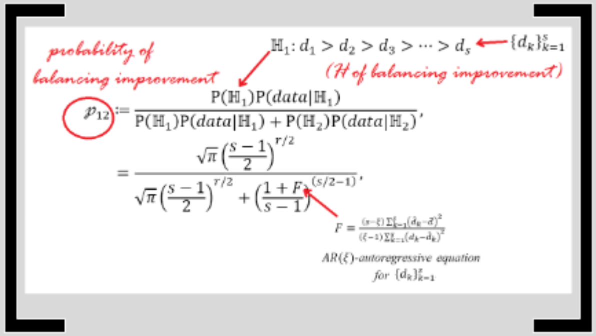

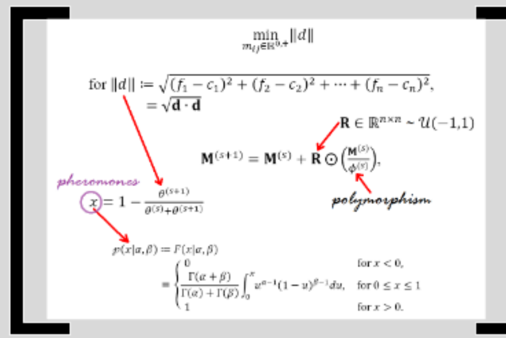

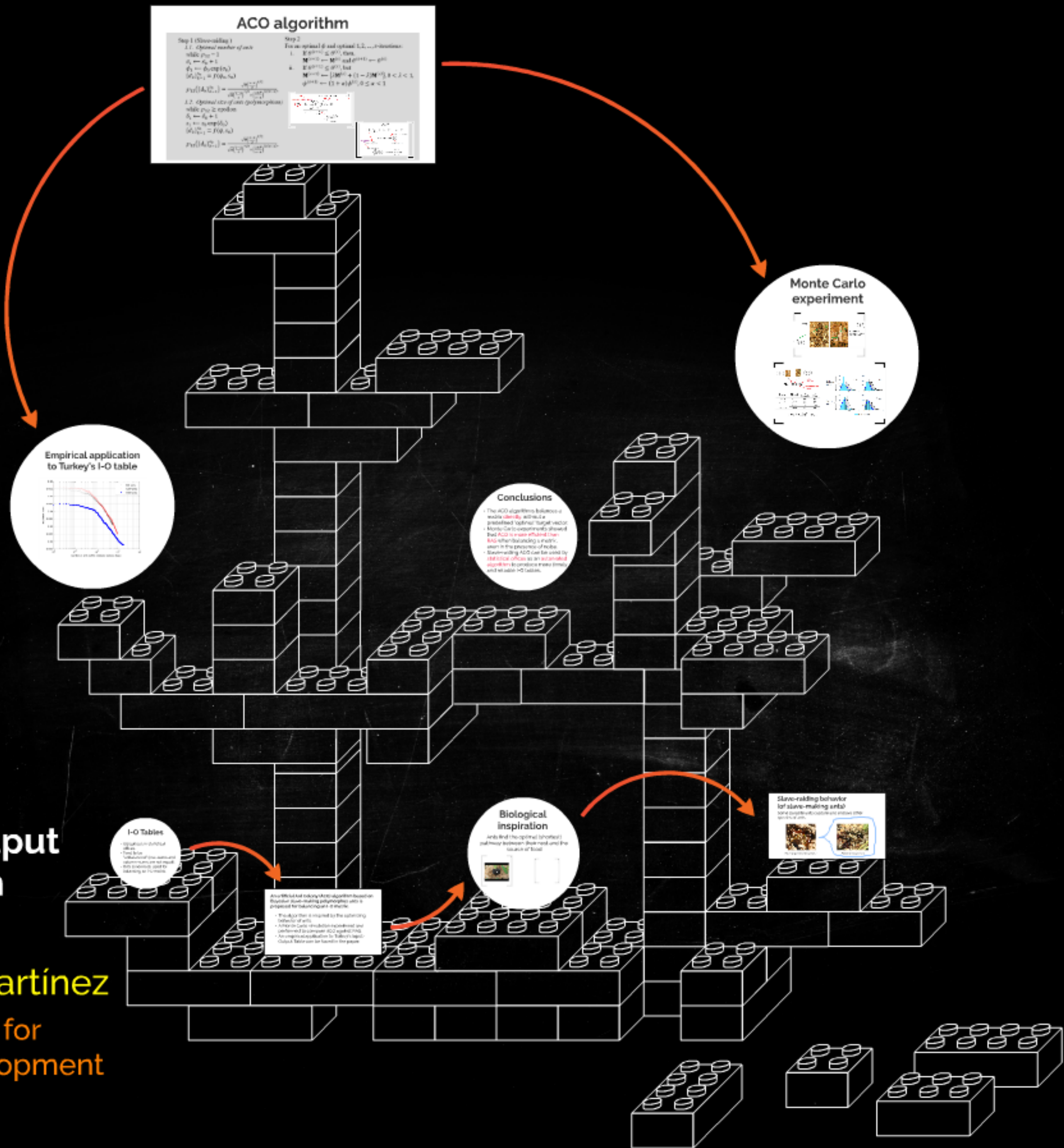

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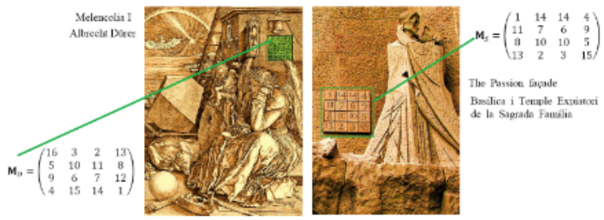
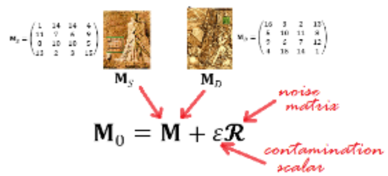
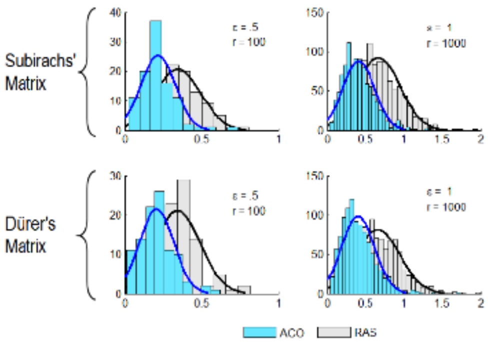
Monte Carlo experiment



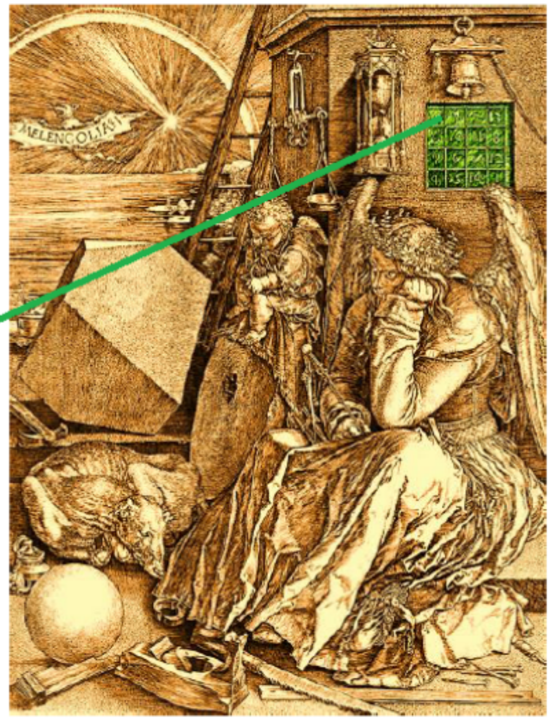


Design	Method	γ Subirachs	γ Dürer
ε = .5	RAS	.3445	.3422
r = 100	ACO	.2154	.2057
ε = 1	RAS	.6678	.6587
r = 1000	ACO	.4113	.4031

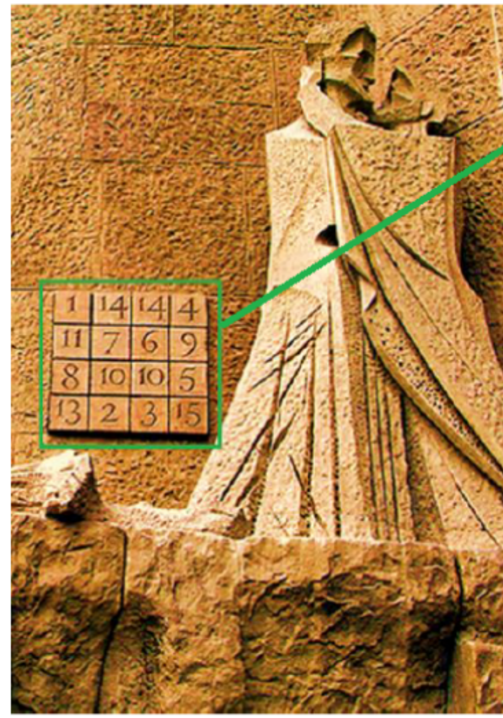
$$\gamma = i^{-2} \sum_{ij} |m_{ij}^0 - m_{ij}|$$



Melencolia I
Albrecht Dürer



$$\mathbf{M}_D = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}$$



$$\mathbf{M}_S = \begin{pmatrix} 1 & 14 & 14 & 4 \\ 11 & 7 & 6 & 9 \\ 8 & 10 & 10 & 5 \\ 13 & 2 & 3 & 15 \end{pmatrix}$$

The Passion façade

Basílica i Temple Expiatori de la Sagrada Família

$$\mathbf{M}_S = \begin{pmatrix} 1 & 14 & 14 & 4 \\ 11 & 7 & 6 & 9 \\ 8 & 10 & 10 & 5 \\ 13 & 2 & 3 & 15 \end{pmatrix}$$

$\mathbf{M}_S$

$\mathbf{M}_D$

$$\mathbf{M}_D = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}$$

$$\mathbf{M}_0 = \mathbf{M} + \varepsilon\mathcal{R}$$

noise matrix

contamination scalar

Design	Method	γ Subirachs	γ Dürer
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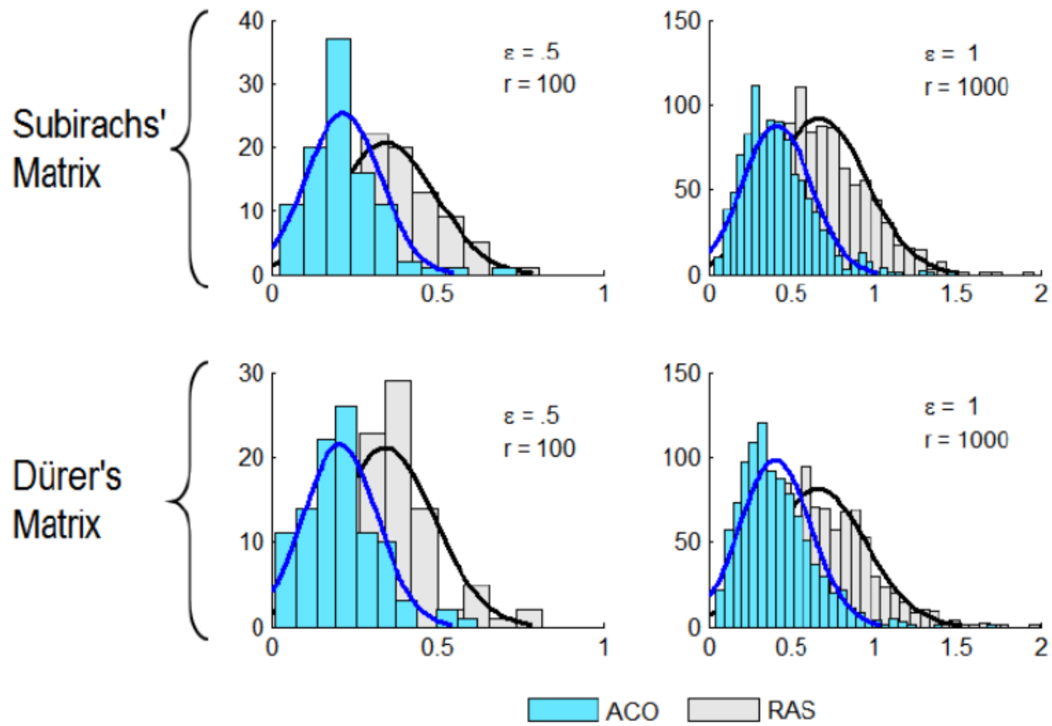
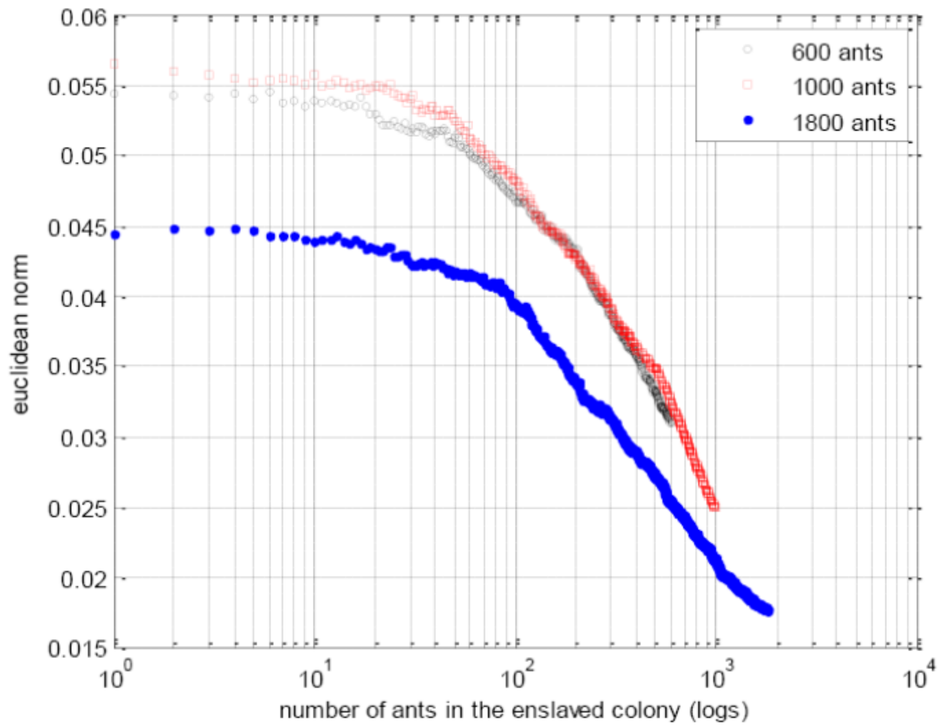
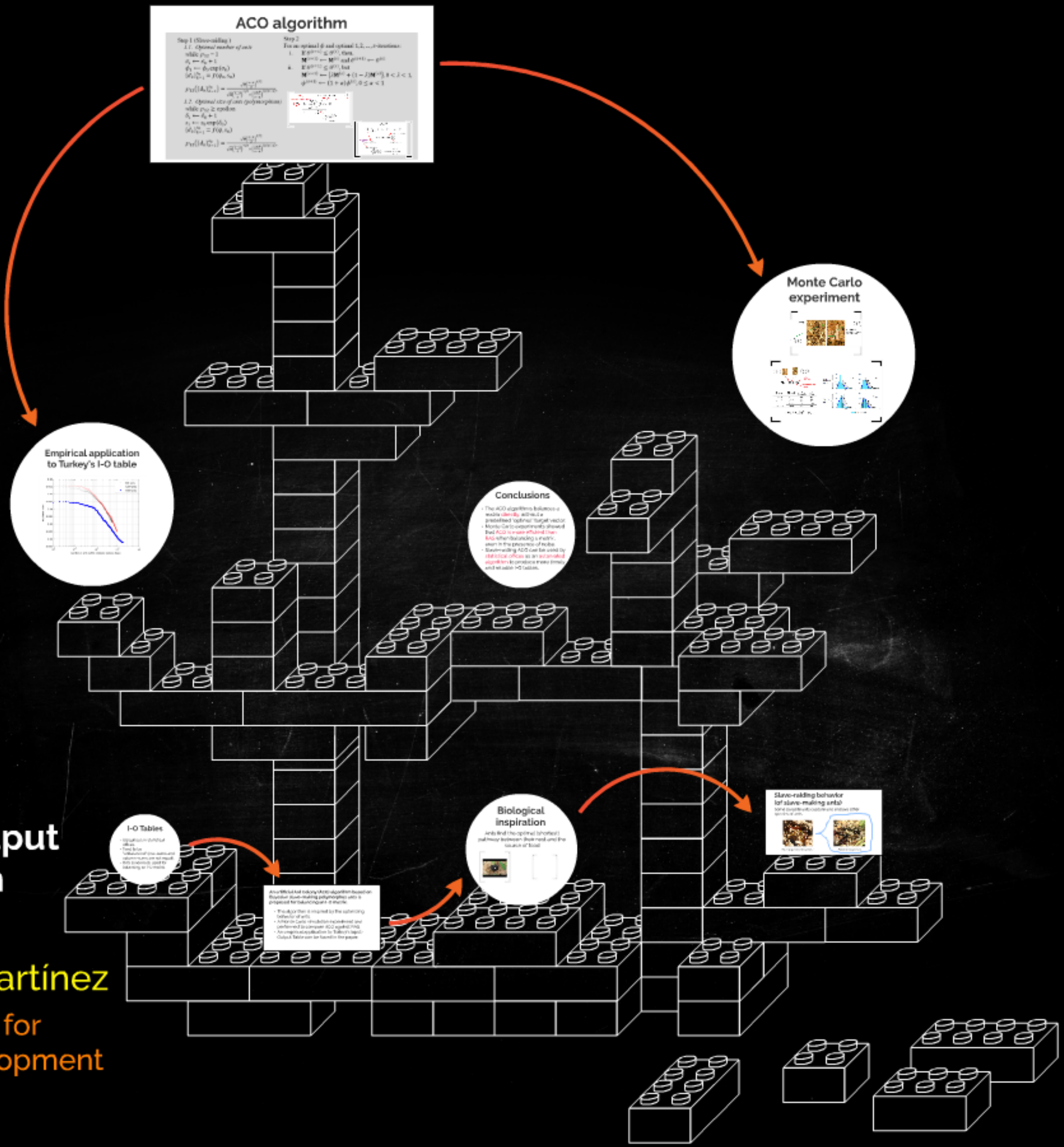


Empirical application to Turkey's I-O table



Conclusions

- The ACO algorithms balances a matrix directly, without a predefined "optimal" target vector.
- Monte Carlo experiments showed that ACO is more efficient than RAS when balancing a matrix, even in the presence of noise.
- Slave-raiding ACO can be used by statistical offices as an automated algorithm to produce more timely and reliable I-O tables.



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