

I-O Tables

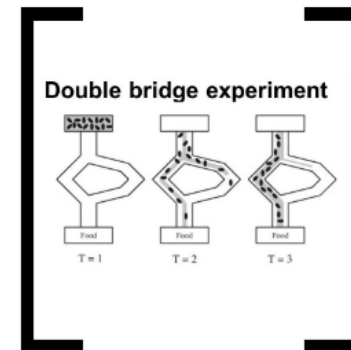
- Ubiquitous in statistical offices.
- Tend to be "unbalanced" (row-sums and column-sums are not equal).
- RAS is normally used for balancing an I-O matrix.

An artificial Ant Colony (ACO) algorithm based on Bayesian slave-making polymorphus ants is proposed for balancing an I-O matrix.

- The algorithm is inspired by the optimizing behavior of ants.
- A Monte Carlo simulation experiment was performed to compare ACO against RAS.
- An empirical application to Turkey's Input-Output Table can be found in the paper.

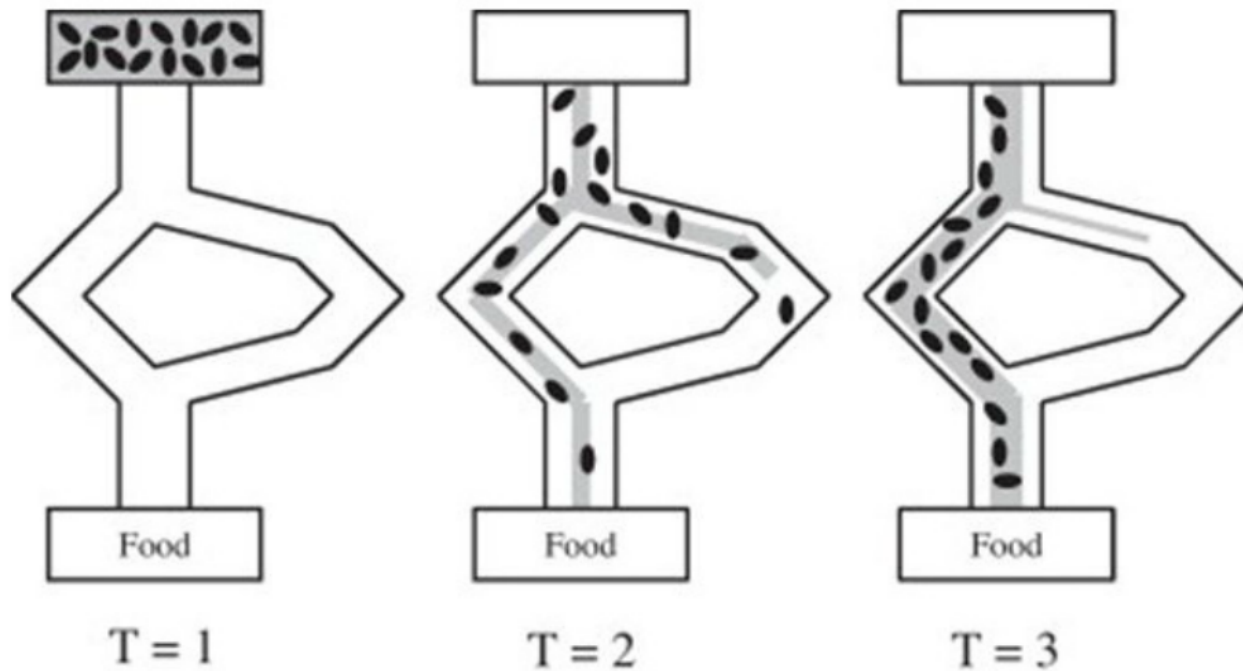
Biological inspiration

Ants find the optimal (shortest)
pathway between their nest and the
source of food





Double bridge experiment



Slave-raiding behavior (of slave-making ants)

Some parasite ants capture and enslave other species of ants



Rossomyrmex minuchae



Proformica ferreri

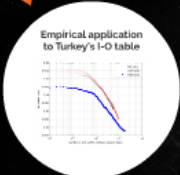
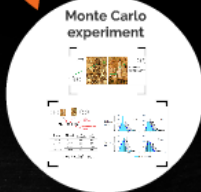
ACO algorithm

```

Step 1 (Initialization)
1.1. Assign number of ants
while stop = 0
  A = 0
  for i = 1 to n
    g(i) = 0
  end for
  g(i) = 1/n
  g(i) = 1/n
end for
rho(A,C) = 1
rho(A,C) = 1

Step 2
for m = 1 to m_max
  for i = 1 to n
    for j = 1 to n
      if rand < rho(A,C)
        g(i) = g(i) + 1/n
      end if
    end for
  end for
  rho(A,C) = 1
end for

```



Conclusions

The ACO algorithm is able to find the optimal solution for the I-O table balancing problem. The algorithm is able to find the optimal solution for the I-O table balancing problem. The algorithm is able to find the optimal solution for the I-O table balancing problem.

I-O Tables

Input-Output tables are a key tool for analyzing the economic structure of a country. They provide a detailed view of the flows of goods and services between different sectors of the economy.

Biological inspiration

The ACO algorithm is inspired by the foraging behavior of ants. Ants find the shortest path to food by leaving pheromone trails. The algorithm mimics this process to find the optimal solution for the I-O table balancing problem.

Slave-raiding behavior of slave-making ants

Slave-making ants are a type of ant that captures and enslaves workers of other ant colonies. This behavior is used as a metaphor for the ACO algorithm, where ants are represented by the algorithm's search process.

Balancing Input-Output tables with Bayesian Slave-raiding Ants

Rolando Gonzales Martínez



Bayesian Institute for Research & Development

ACO algorithm

Step 1 (Slave-raiding)

1.1. Optimal number of ants

while $p_{12} = 1$

$\sigma_1 \leftarrow \sigma_0 + 1$

$\phi_1 \leftarrow \phi_0 \exp(\sigma_0)$

$\{d_k\}_{k=1}^{s_0} = f(\phi_0, s_0)$

$$p_{12}(\{d_k\}_{k=1}^{s_0}) = \frac{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)},}$$

1.2. Optimal size of ants (polymorphism)

while $p_{12} \geq \epsilon$

$\delta_1 \leftarrow \delta_0 + 1$

$s_1 \leftarrow s_0 \exp(\delta_0)$

$\{d_k\}_{k=1}^{s_0} = f(\phi, s_0)$

$$p_{12}(\{d_k\}_{k=1}^{s_0}) = \frac{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)},}$$

Step 2

For an optimal ϕ and optimal 1,2, ..., s-iterations:

i. If $\theta^{(s+1)} \leq \theta^{(s)}$, then,

$\mathbf{M}^{(s+1)} \leftarrow \mathbf{M}^{(s)}$ and $\theta^{(s+1)} \leftarrow \theta^{(s)}$

ii. If $\theta^{(s+1)} > \theta^{(s)}$, but

$\mathbf{M}^{(s+1)} \leftarrow \{\lambda \mathbf{M}^{(s)} + (1 - \lambda) \mathbf{M}^{(s)}\}, 0 < \lambda < 1,$

$\phi^{(s+1)} \leftarrow (1 + \alpha) \phi^{(s)}, 0 \leq \alpha < 1$

probability of balancing improvement $H_1: d_1 > d_2 > d_3 > \dots > d_s \leftarrow (d_k)_{k=1}^s$ (H of balancing improvement)

$$p_{12} = \frac{P(H_1)P(\text{data}|H_1)}{P(H_1)P(\text{data}|H_1) + P(H_2)P(\text{data}|H_2)}$$

$$= \frac{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)},}$$

$F = \frac{(s-1) \sum_{k=1}^s (d_k - \bar{d})^2}{\sum_{k=1}^s (d_k - \bar{d})^2}$
ANOVA-analytic equation for $(d_k)_{k=1}^s$

$\min_{\mathbf{M} \in \mathbb{R}^{n \times n}} \|\mathbf{d}\|$

for $\|\mathbf{d}\| = \sqrt{(f_1 - c_1)^2 + (f_2 - c_2)^2 + \dots + (f_n - c_n)^2} = \sqrt{\mathbf{d}^T \mathbf{d}}$

$\mathbf{R} \in \mathbb{R}^{n \times n} - \mathbf{U}(-1,1)$

$\mathbf{M}^{(s+1)} = \mathbf{M}^{(s)} + \mathbf{R} \odot \left(\frac{\mathbf{M}^{(s)}}{\|\mathbf{M}^{(s)}\|}\right)$

phenomenon $\lambda = 1 - \frac{\theta^{(s+1)}}{\theta^{(s)}}$ polymorphism

$p(x, \beta) = \begin{cases} \frac{\Gamma(x+\beta)}{\Gamma(x)\Gamma(\beta)} u^{x-1}(1-u)^{\beta-1} du & \text{for } x < 0 \\ \frac{\Gamma(x+\beta)}{\Gamma(x)\Gamma(\beta)} u^{x-1}(1-u)^{\beta-1} du & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } x > 0 \end{cases}$

probability of
balancing improvement

$$\mathbb{H}_1: d_1 > d_2 > d_3 > \dots > d_s \leftarrow \{d_k\}_{k=1}^s$$

(\mathbb{H} of balancing improvement)

$$p_{12} := \frac{P(\mathbb{H}_1)P(\text{data}|\mathbb{H}_1)}{P(\mathbb{H}_1)P(\text{data}|\mathbb{H}_1) + P(\mathbb{H}_2)P(\text{data}|\mathbb{H}_2)},$$

$$= \frac{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)},}$$

$$F = \frac{(s-\xi) \sum_{k=1}^s (\hat{d}_k - \bar{d})^2}{(\xi-1) \sum_{k=1}^s (d_k - \hat{d}_k)^2}$$

AR(ξ)-autoregressive equation
for $\{d_k\}_{k=1}^s$

$$\min_{m_{ij} \in \mathbb{R}^{0,+}} \|d\|$$

$$\text{for } \|d\| := \sqrt{(f_1 - c_1)^2 + (f_2 - c_2)^2 + \dots + (f_n - c_n)^2},$$

$$= \sqrt{\mathbf{d} \cdot \mathbf{d}}$$

$$\mathbf{R} \in \mathbb{R}^{n \times n} \sim \mathcal{U}(-1,1)$$

$$\mathbf{M}^{(s+1)} = \mathbf{M}^{(s)} + \mathbf{R} \odot \left(\frac{\mathbf{M}^{(s)}}{\phi^{(s)}} \right),$$

pheromones

$$x = 1 - \frac{\theta^{(s+1)}}{\theta^{(s)} + \theta^{(s+1)}}$$

polymorphism

$$p(x|\alpha, \beta) := F(x|\alpha, \beta)$$

$$= \begin{cases} 0 & \text{for } x < 0, \\ \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) + \Gamma(\beta)} \int_0^x u^{\alpha-1} (1-u)^{\beta-1} du, & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } x > 0. \end{cases}$$

ACO algorithm

Step 1 (Slave-raiding)

1.1. Optimal number of ants

while $p_{12} = 1$

$\sigma_1 \leftarrow \sigma_0 + 1$

$\phi_1 \leftarrow \phi_0 \exp(\sigma_0)$

$\{d_k\}_{k=1}^{s_0} = f(\phi_0, s_0)$

$$p_{12}(\{d_k\}_{k=1}^{s_0}) = \frac{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)},}$$

1.2. Optimal size of ants (polymorphism)

while $p_{12} \geq \text{epsilon}$

$\delta_1 \leftarrow \delta_0 + 1$

$s_1 \leftarrow s_0 \exp(\delta_0)$

$\{d_k\}_{k=1}^{s_0} = f(\phi, s_0)$

$$p_{12}(\{d_k\}_{k=1}^{s_0}) = \frac{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)},}$$

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probability of balancing improvement $\mathbb{H}_1: d_1 > d_2 > d_3 > \dots > d_s \leftarrow \{d_k\}_{k=1}^s$ (H of balancing improvement)

$$p_{12} = \frac{P(\mathbb{H}_1)P(\text{data}|\mathbb{H}_1)}{P(\mathbb{H}_1)P(\text{data}|\mathbb{H}_1) + P(\mathbb{H}_2)P(\text{data}|\mathbb{H}_2)}$$

$$= \frac{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2}}{\sqrt{\pi} \left(\frac{s-1}{2}\right)^{r/2} + \left(\frac{1+F}{s-1}\right)^{(s/2-1)},}$$

$F = \frac{(s-1) \sum_{k=1}^s (d_k - \bar{d})^2}{(s-1) \sum_{k=1}^s (d_k - d_1)^2}$

AR(ξ)-autoregressive equation for $\{d_k\}_{k=1}^s$

$\min_{\mathbf{M} \in \mathbb{R}^{n \times n}} \|\mathbf{d}\|$

for $\|\mathbf{d}\| := \sqrt{(f_1 - c_1)^2 + (f_2 - c_2)^2 + \dots + (f_n - c_n)^2}$
 $= \sqrt{\mathbf{d} \cdot \mathbf{d}}$

$\mathbf{R} \in \mathbb{R}^{n \times n} - \mathcal{U}(-1,1)$

$\mathbf{M}^{(s+1)} = \mathbf{M}^{(s)} + \mathbf{R} \odot \left(\frac{\mathbf{M}^{(s)}}{\|\mathbf{M}^{(s)}\|}\right)$

pheromones $\alpha = 1 - \frac{\theta^{(s+1)}}{\theta^{(s)} + \theta^{(s+1)}}$ polymorphism

$p(x|\alpha, \beta) = \Gamma(x|\alpha, \beta)$

$$= \begin{cases} 0 & \text{for } x < 0, \\ \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} \int_0^x u^{\alpha-1} (1-u)^{\beta-1} du, & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } x > 0. \end{cases}$$

Monte Carlo experiment

Melencolia I
 Albrecht Dürer

$M_0 = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}$

$M_1 = \begin{pmatrix} 1 & 14 & 14 & 4 \\ 11 & 7 & 6 & 9 \\ 8 & 10 & 10 & 5 \\ 13 & 2 & 3 & 15 \end{pmatrix}$

The Passion Epistle
 Basílica i Temple Externi de la Sagrada Família

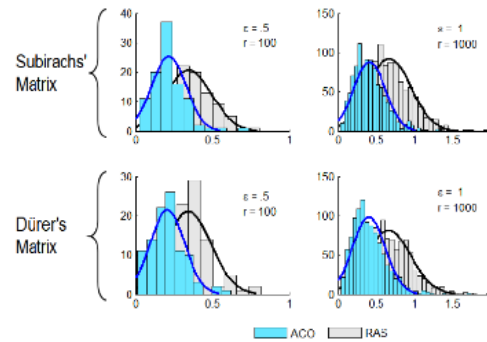
$M_0 = M + \epsilon R$

noise matrix
contamination scalar

$M_0 = \begin{pmatrix} 1 & 14 & 14 & 4 \\ 11 & 7 & 6 & 9 \\ 8 & 10 & 10 & 5 \\ 13 & 2 & 3 & 15 \end{pmatrix} + \epsilon \begin{pmatrix} 15 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}$

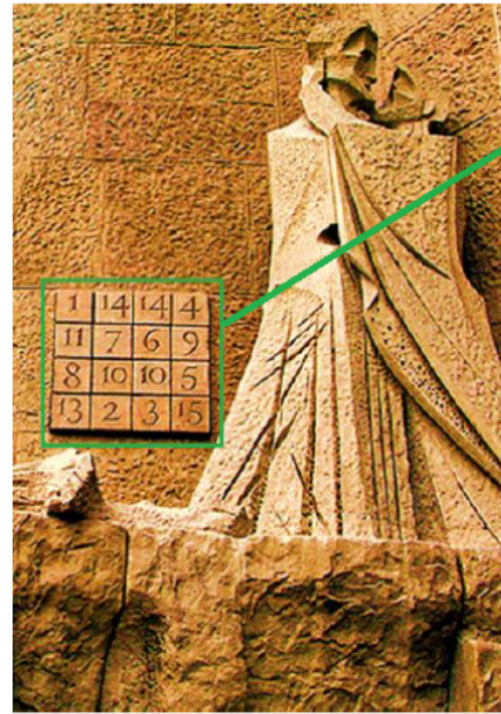
Design	Method	γ Subirachs	γ Dürer
$\epsilon = .5$	RAS	.3445	.3422
$r = 100$	ACO	.2154	.2057
$\epsilon = 1$	RAS	.6678	.6587
$r = 1000$	ACO	.4113	.4031

$$\gamma = i^{-2} \sum_{ij} |m_{ij}^0 - m_{ij}|$$



Melencolia I
Albrecht Dürer

$$M_D = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}$$



$$M_S = \begin{pmatrix} 1 & 14 & 14 & 4 \\ 11 & 7 & 6 & 9 \\ 8 & 10 & 10 & 5 \\ 13 & 2 & 3 & 15 \end{pmatrix}$$

The Passion façade
Basilica i Temple Expiatori
de la Sagrada Família

$$M_S = \begin{pmatrix} 1 & 14 & 14 & 4 \\ 11 & 7 & 6 & 9 \\ 8 & 10 & 10 & 5 \\ 13 & 2 & 3 & 15 \end{pmatrix}$$



M_S



M_D

$$M_D = \begin{pmatrix} 16 & 3 & 2 & 13 \\ 5 & 10 & 11 & 8 \\ 9 & 6 & 7 & 12 \\ 4 & 15 & 14 & 1 \end{pmatrix}$$

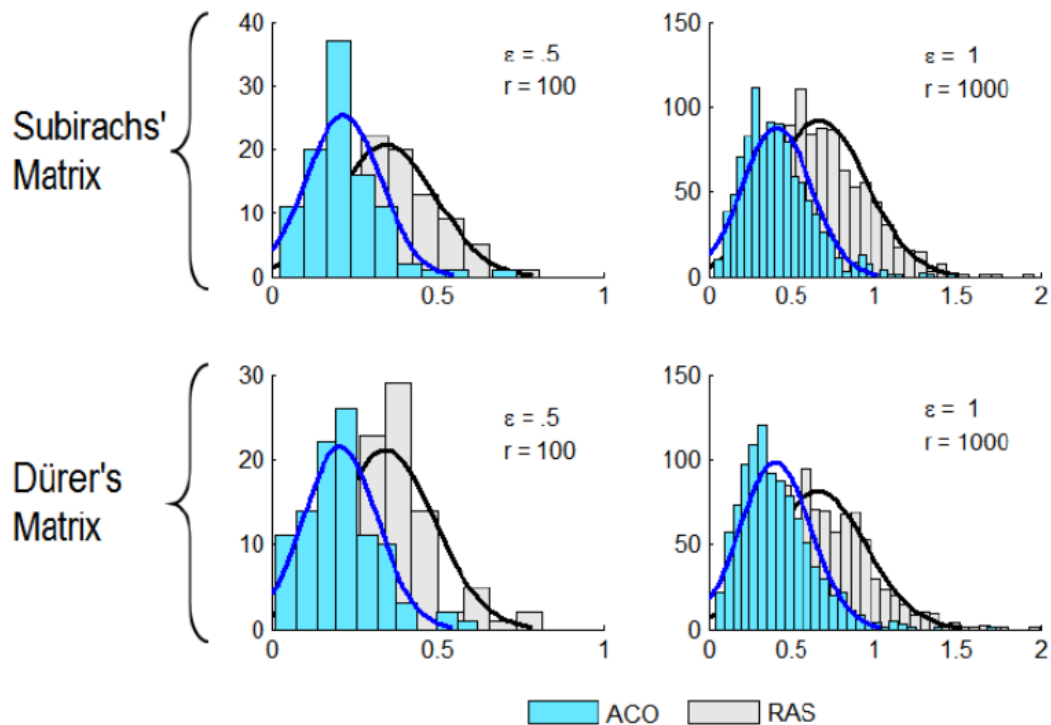
$$M_0 = M + \epsilon R$$

noise matrix

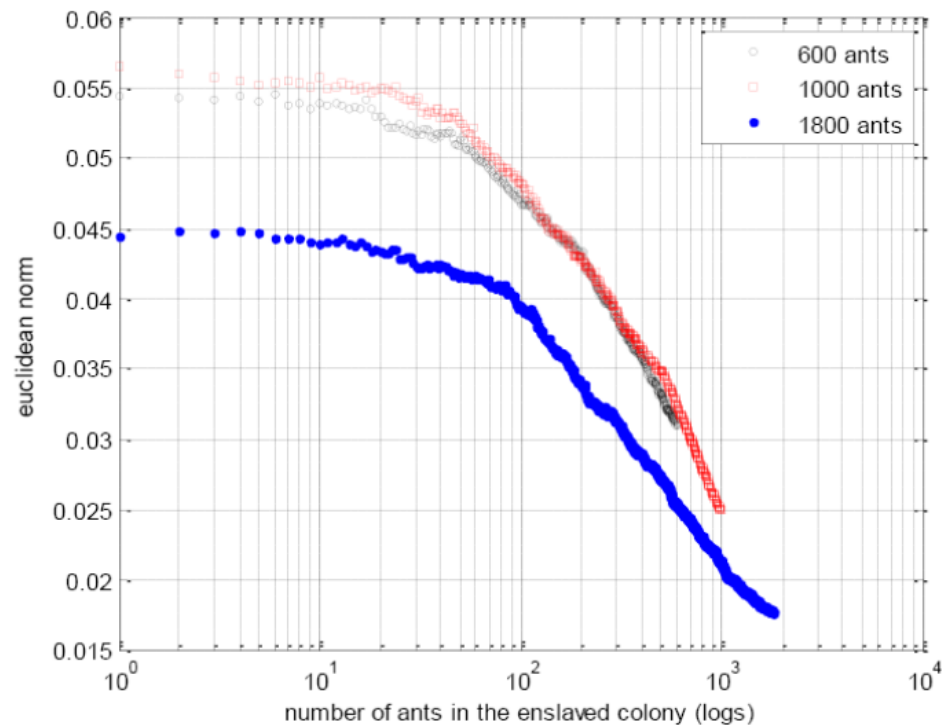
contamination scalar

Design	Method	γ Subirachs	γ Dürer
$\epsilon = .5$	RAS	.3445	.3422
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$$\gamma = i^{-2} \sum_{ij} |m_{ij}^0 - m_{ij}|$$



Empirical application to Turkey's I-O table



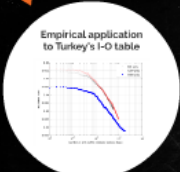
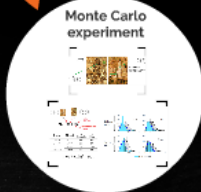
Conclusions

- The ACO algorithms balances a matrix **directly**, without a predefined "optimal" target vector.
- Monte Carlo experiments showed that **ACO is more efficient than RAS** when balancing a matrix, even in the presence of noise.
- Slave-raiding ACO can be used by **statistical offices** as an **automated algorithm** to produce more timely and reliable I-O tables.

ACO algorithm

```

Step 1 (Initialization)
1.1. Assign number of ants
while stop = 0
  A = 0
  B = 0
  C = 0
  D = 0
  E = 0
  F = 0
  G = 0
  H = 0
  I = 0
  J = 0
  K = 0
  L = 0
  M = 0
  N = 0
  O = 0
  P = 0
  Q = 0
  R = 0
  S = 0
  T = 0
  U = 0
  V = 0
  W = 0
  X = 0
  Y = 0
  Z = 0
  AA = 0
  AB = 0
  AC = 0
  AD = 0
  AE = 0
  AF = 0
  AG = 0
  AH = 0
  AI = 0
  AJ = 0
  AK = 0
  AL = 0
  AM = 0
  AN = 0
  AO = 0
  AP = 0
  AQ = 0
  AR = 0
  AS = 0
  AT = 0
  AU = 0
  AV = 0
  AW = 0
  AX = 0
  AY = 0
  AZ = 0
  BA = 0
  BB = 0
  BC = 0
  BD = 0
  BE = 0
  BF = 0
  BG = 0
  BH = 0
  BI = 0
  BJ = 0
  BK = 0
  BL = 0
  BM = 0
  BN = 0
  BO = 0
  BP = 0
  BQ = 0
  BR = 0
  BS = 0
  BT = 0
  BU = 0
  BV = 0
  BW = 0
  BX = 0
  BY = 0
  BZ = 0
  CA = 0
  CB = 0
  CC = 0
  CD = 0
  CE = 0
  CF = 0
  CG = 0
  CH = 0
  CI = 0
  CJ = 0
  CK = 0
  CL = 0
  CM = 0
  CN = 0
  CO = 0
  CP = 0
  CQ = 0
  CR = 0
  CS = 0
  CT = 0
  CU = 0
  CV = 0
  CW = 0
  CX = 0
  CY = 0
  CZ = 0
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  DB = 0
  DC = 0
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  DJ = 0
  DK = 0
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  FE = 0
  FF = 0
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  FH = 0
  FI = 0
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  FL = 0
  FM = 0
  FN = 0
  FO = 0
  FP = 0
  FQ = 0
  FR = 0
  FS = 0
  FT = 0
  FU = 0
  FV = 0
  FW = 0
  FX = 0
  FY = 0
  FZ = 0
  GA = 0
  GB = 0
  GC = 0
  GD = 0
  GE = 0
  GF = 0
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  GK = 0
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  GN = 0
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  GQ = 0
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  GT = 0
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  GZ = 0
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  HF = 0
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  HI = 0
  HJ = 0
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  HO = 0
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  HZ = 0
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  IL = 0
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  IN = 0
  IO = 0
  IP = 0
  IQ = 0
  IR = 0
  IS = 0
  IT = 0
  IU = 0
  IV = 0
  IW = 0
  IX = 0
  IY = 0
  IZ = 0
  JA = 0
  JB = 0
  JC = 0
  JD = 0
  JE = 0
  JF = 0
  JG = 0
  JH = 0
  JI = 0
  JJ = 0
  JK = 0
  JL = 0
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  KD = 0
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  KG = 0
  KH = 0
  KI = 0
  KJ = 0
  KL = 0
  KM = 0
  KN = 0
  KO = 0
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  KQ = 0
  KR = 0
  KS = 0
  KT = 0
  KU = 0
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  KW = 0
  KX = 0
  KY = 0
  KZ = 0
  LA = 0
  LB = 0
  LC = 0
  LD = 0
  LE = 0
  LF = 0
  LG = 0
  LH = 0
  LI = 0
  LJ = 0
  LK = 0
  LL = 0
  LM = 0
  LN = 0
  LO = 0
  LP = 0
  LQ = 0
  LR = 0
  LS = 0
  LT = 0
  LU = 0
  LV = 0
  LW = 0
  LX = 0
  LY = 0
  LZ = 0
  MA = 0
  MB = 0
  MC = 0
  MD = 0
  ME = 0
  MF = 0
  MG = 0
  MH = 0
  MI = 0
  MJ = 0
  MK = 0
  ML = 0
  MM = 0
  MN = 0
  MO = 0
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  MW = 0
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  NG = 0
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  NJ = 0
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  NL = 0
  NM = 0
  NN = 0
  NO = 0
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  NX = 0
  NY = 0
  NZ = 0
  OA = 0
  OB = 0
  OC = 0
  OD = 0
  OE = 0
  OF = 0
  OG = 0
  OH = 0
  OI = 0
  OJ = 0
  OK = 0
  OL = 0
  OM = 0
  ON = 0
  OO = 0
  OP = 0
  OQ = 0
  OR = 0
  OS = 0
  OT = 0
  OU = 0
  OV = 0
  OW = 0
  OX = 0
  OY = 0
  OZ = 0
  PA = 0
  PB = 0
  PC = 0
  PD = 0
  PE = 0
  PF = 0
  PG = 0
  PH = 0
  PI = 0
  PJ = 0
  PK = 0
  PL = 0
  PM = 0
  PN = 0
  PO = 0
  PP = 0
  PQ = 0
  PR = 0
  PS = 0
  PT = 0
  PU = 0
  PV = 0
  PW = 0
  PX = 0
  PY = 0
  PZ = 0
  QA = 0
  QB = 0
  QC = 0
  QD = 0
  QE = 0
  QF = 0
  QG = 0
  QH = 0
  QI = 0
  QJ = 0
  QK = 0
  QL = 0
  QM = 0
  QN = 0
  QO = 0
  QP = 0
  QQ = 0
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  QS = 0
  QT = 0
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  QV = 0
  QW = 0
  QX = 0
  QY = 0
  QZ = 0
  RA = 0
  RB = 0
  RC = 0
  RD = 0
  RE = 0
  RF = 0
  RG = 0
  RH = 0
  RI = 0
  RJ = 0
  RK = 0
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```



Conclusions

The ACO algorithm is able to find the optimal solution for the I-O table balancing problem. The algorithm is able to find the optimal solution for the I-O table balancing problem. The algorithm is able to find the optimal solution for the I-O table balancing problem.

I-O Tables

Input-Output tables are a key component of national accounts. They provide a detailed view of the economic structure of a country, showing the flows of goods and services between different sectors of the economy.

Biological Inspiration

The ACO algorithm is inspired by the behavior of ants. Ants find the shortest path to food by leaving pheromone trails. The algorithm mimics this process to find the optimal solution for the I-O table balancing problem.

Slave-raiding behavior of slave-making ants

Slave-making ants are a type of ant that captures and enslaves workers from other ant colonies. This behavior is used as a metaphor for the ACO algorithm, where ants explore different solutions and find the best one.

Balancing Input-Output tables with Bayesian Slave-raiding Ants

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